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THE TURBULENT DIFFUSION OF RIVER CONTAMINANTS

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INTRODUCTIONS

Considerable work has been done (1,2,3,4,5) toward the derivation of the basic mathematical relationships describing the turbulent diffusion of heat and of suspended material in liquid and gas streams. However, the developed relationships are generally applicable only to ideal stream and source conditions and thus cannot be directly applied in defining the diffusion pattern of contaminants in the Columbia River. The objective of this study was to obtain a workable mathematical relationship, in agreement with all available data, which would define the existing turbulent diffusion pattern of reactor cooling water in the Columbia River thus allowing the prediction of river temperatures and radioactivity concentrations under proposed operating conditions.

SUMMARY AND CONCLUSIONS

Both a theoretical and an empirical approach were pursued toward defining the turbulent diffusion process encountered in the Columbia River. The merits and limitations of each approach are discussed. Although no solution was obtained for the differential equation encountered in the theoretical development, its derivation is included for the benefit of those interested in further study of this or a similar problem. The empirical approach is completely outlined and gives an adequate fit to the available data. However, it cannot be extended to conditions not defined by the data without additional experimental work which is outlined.

DISCUSSION

1. The Statement of the Problem

As water from the Columbia River passes through the HAPO nuclear reactors, in addition to absorbing heat generated by the controlled fission process, the impurities in the water become radioactive through irradiation. Following a retention period to allow cooling and radioactive decay, the water is discharged into the river through submerged pipes. Once in the river, the residual heat and radioactivity is dispersed by turbulent diffusion and downstream conveyance. The rate and pattern of this dispersion are of prime concern due to the effect of the heat on the aquatic life in the river and on the productive capacity of downstream reactors. Of equal concern is the effect of the residual radioactivity on the aquatic life and on the populous relying on the Columbia River for drinking water supplies. For these reasons it was imperative that an effort be made to define the turbulent diffusion of this residual heat and radioactivity.

In considering this problem, it is assumed that the diffusion process is significant with respect to the other variables only in the cross-river direction. The vertical diffusion is assumed to be negligible both because of the relatively small vertical dimension as compared to the horizontal dimension and also because of the source condition, i.e., the water is discharged into the river at the bed level with sufficient pressure to cause its distribution throughout the entire vertical line at that point. Diffusion along the direction of flow can be neglected if radioactive decay is considered separately and if the observed diffusion is attributed to transverse turbulence. If it is also assumed that the absorption of heat or radioactivity by the river bed and by the atmosphere is negligible and that there is a true line source of heat and radioactivity, the following boundary conditions are established:

A. There exists a delta-function at the source cross-section of the river, i.e., there is zero radioactivity at every point except the source point where its concentration is infinite.

B. There is a uniform distribution of activity at an infinite distance downstream from the source and the concentration is equal to the rate of activity discharge at the source divided by the river flow rate.

2. The Normalizing Variable

The mathematical analysis requires some variable whose range is constant for all river cross-sections. The basic variable of linear distance could, therefore, not be used. Other investigators have used the fraction of total distance but even this is not suitable due to the extreme variability in the contour of the river bed. It has been suggested [5] that a fraction of flow variable be used. This variable signifies the fraction of the total river flow passing between a given point and reference bank. The use of this variable essentially eliminates islands and shelfs from the apparent contour of the river bed.

3. The Theoretical Derivation

A theoretical development is certainly preferred since it would embody the physical laws operative in the turbulent diffusion process. Unfortunately, this process is not well defined in terms of basic physical laws; however, certain analogies may be drawn between this and other transfer processes.

Notation:

C - concentration of contaminants at any point in the river

x - direction of principle flow vector

φ - cross-river coordinate in units of fraction of flow between any point and the reference shore

z - cross-river coordinate in units of fraction of total width

$K(\varphi)$ - coefficient of diffusion (as a function of φ)

u_x - river velocity in the downstream direction

ρ - density of the river water

τ_0 - shearstress at the bank of the river

τ - shearstress at any point in the river

The relationship defining the velocity profile across the river is assumed to be

$$u_x = \beta(\varphi - \varphi^2)^{\mu} \qquad 0 \leq \varphi \leq 1$$

where β and μ are empirical constants. The coefficient of eddy viscosity (coefficient for turbulent momentum transfer) is given by Brunt[6] for the case of the atmosphere. In considering the diffusion of matter, it has been assumed[7] that the coefficient of eddy diffusivity (coefficient for turbulent mass transfer) is equal to the eddy viscosity coefficient. Assuming that these same laws are applicable in the case of incompressible fluid flow, it follows that

$$\tau = \rho K(\varphi) \frac{\partial u_x}{\partial \varphi}$$

Wang[8] points out that for turbulent flow in channels of constant cross-section,

$$\tau = \tau_0 (1 - 2z) \qquad 0 \le z \le 1/2$$
$$\tau = \tau_0 (2z - 1) \qquad 1/2 \le z \le 1$$

If it may be assumed for case of turbulent flow in river channels that

$$\tau = \tau_0 (1 - 2\varphi) \qquad 0 \le \varphi \le 1/2$$
$$\tau = \tau_0 (2\varphi - 1) \qquad 1/2 \le \varphi \le 1$$

then it is easily shown that

$$K(\varphi) = \frac{\tau_0}{\rho \beta \mu} (\varphi - \varphi^2)^{1-\mu} \qquad 0 \le \varphi \le 1$$

A differential equation defining the diffusion process is[7]

$$u_x \frac{\partial c}{\partial x} = \frac{\partial}{\partial \varphi}\left[K(\varphi) \frac{\partial c}{\partial \varphi} \right]$$

Using the relationships thus far derived, this expression becomes

$$\beta (\varphi - \varphi^2)^{\mu} \frac{\partial c}{\partial x} = \frac{\partial}{\partial \varphi}\left[\frac{\tau_0}{\rho \beta \mu} (\varphi - \varphi^2)^{1-\mu} \frac{\partial c}{\partial \varphi} \right]$$

which upon expanding and rearranging is found to be

$$\frac{\rho \beta^2 \mu}{\tau_0} \frac{\partial c}{\partial x} = (\varphi - \varphi^2)^{1-2\mu} \frac{\partial^2 c}{\partial \varphi^2} + (1-\mu)(1-2\varphi)(\varphi - \varphi^2)^{-2\mu} \frac{\partial c}{\partial \varphi}$$

The variables are therefore separable and if a solution of the form

$$C = \eta(x)\, y(\varphi)$$

is assumed, then the following differential equations must be solved

$$\frac{\rho \beta^2 \mu}{\tau_0} \frac{\eta'}{\eta} = -\lambda^2$$

$$(\varphi - \varphi^2)^{1-2\mu} y'' + (1-\mu)(1-2\varphi)(\varphi - \varphi^2)^{-2\mu} y' + \lambda^2 y = 0$$

No solution has been found for the latter differential equation. If this expression were solved, its applicability would be limited by its probable complexity and the uncertainty of the underlying assumptions. Further, the units of the answer — radioactivity per ($cm^2 \times \varphi$) — would require a conversion between φ and centimeters. Such a relationship cannot be defined due to its dependence on the cross-sectional profile of the river.

4. The Empirical Derivation

This type of problem suggests that some statistical probability function or an adaptation therof might be used to define the actual conditions since the distribution of radioactivity and heat is being considered and this distribution is continually being altered by the random movement of radioactive and thermally active molecules under the influence of certain forces. The probability function must have for its limits a delta-function and a uniform distribution and must be completely represented in a bounded interval. An obvious selection is the beta probability function. This choice is corroborated by the analysis of the available data by means of statistical moments.(9) As in the case of the theoretical derivation, the fraction of flow variable, φ, is used as a normalizing variable. However, since the relationship is empirical, this variable is not imposed on the units of the concentration. The empirical relationship is

$$\frac{C}{C_o} = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \varphi^{p-1}(1-\varphi)^{q-1}$$

where C_o is the concentration of radioactivity under uniform distribution conditions, and C is the concentration at any point in the river. Expressions must be determined for p and q such that the resulting beta function will fit the boundary conditions and the available data.

Analytically, the mean and variance of the beta probability function are

$$\mu = \frac{p}{p+q}$$

and

$$\sigma^2 = \frac{pq}{(p+q)^2(p+q+1)}$$

Solving the two expressions simultaneously for p and q gives

$$p = \frac{\mu}{\sigma^2}\left[\mu(1-\mu) - \sigma^2\right]$$

$$q = \frac{(1-\mu)}{\sigma^2}\left[\mu(1-\mu) - \sigma^2\right]$$

The parameters p and q were determined from the original data(5) by use of statistical moments.(9) These "observed" values for the parameters were used to obtain "observed" measures of the mean and variance for each traverse point.

Now the mean originates at the point of discharge (in terms of the fraction of flow variable) and must assymptotically approach the midpoint of the river as the diffusing matter proceeds downstream. The "observed" values of the mean were found to be adequately represented by the exponential relationship:

$$\mu = 0.5 - (0.5 - s)e^{-ax}$$

where s is the point of discharge, x is the distance downstream, and a is an empirical constant.

The variance, on the other hand, is zero at the point of discharge and must increase assymptotically to 1/12, the variance of the unit range rectangular distribution. The "observed" values of the variance were found to be best represented by the hyperbolic function

$$\sigma^2 = \left[\frac{b\sigma_M^2 x}{b\sigma_M x + 1}\right]^2$$

where σ_M is the square root of 1/12, x is the distance downstream, and b is an empirical constant.

Unfortunately, it did not appear that the same basic type of relationship, i.e., hyperbolic or exponential, would satisfactorily define both relationships. Ideally, a theoretical basis for the expressions defining the mean and the variance would be preferred; however, a search of the literature failed to provide such a basis.

5. The Empirical Fit

Using the expressions obtained for the mean and variance, corresponding values were computed for the parameters p and q for each traverse point. These "fitted" concentration distributions were plotted along with the "observed" distributions and the individual field measurements and are reproduced in Figures 1-10. An inspection of these graphs indicated a reasonable degree of agreement with the exception of one or two traverses in which the inconsistency between data from adjacent traverses tended to discredit the validity of these measurements. It is believed that the fit obtained by this method is as good as any that could be achieved.

CONCLUSIONS AND RECOMMENDATIONS

Although the expression obtained through the adaptation of the beta probability function was found to adequately represent the available field data, it cannot be extended to conditions not defined by the data due to the empirical nature of the derivation. To obtain a usable relationship, one of the following results must be realized:

1. The theoretical derivation included herein (or some other theoretical derivation) must be completed and checked against the available data.

2. Any experimental studies and/or additional field measurements must provide a broader and more accurate basis for the development using the beta probability function.

3. A different mathematical model must be found which satisfies all of the necessary source and boundary conditions, represents the available data, and has a sufficiently sound theoretical basis to allow its extension to proposed operating conditions.

Of the above methods of attack, the first and the last would require a fresh approach. The second alternative holds promise although the attendant economic factors are considerable. It would seem that the most useable results could be achieved through construction of a water trough or model river of such dimensions that a rough reproduction of the true river turbulence could be attained. Through evaluation of measurements obtained from this trough and a small number of confirmatory measurements in the Columbia River, an applicable solution could be obtained for the problem and much valuable data would be made available which could be used in further studies of the turbulent diffusion phenomenon.

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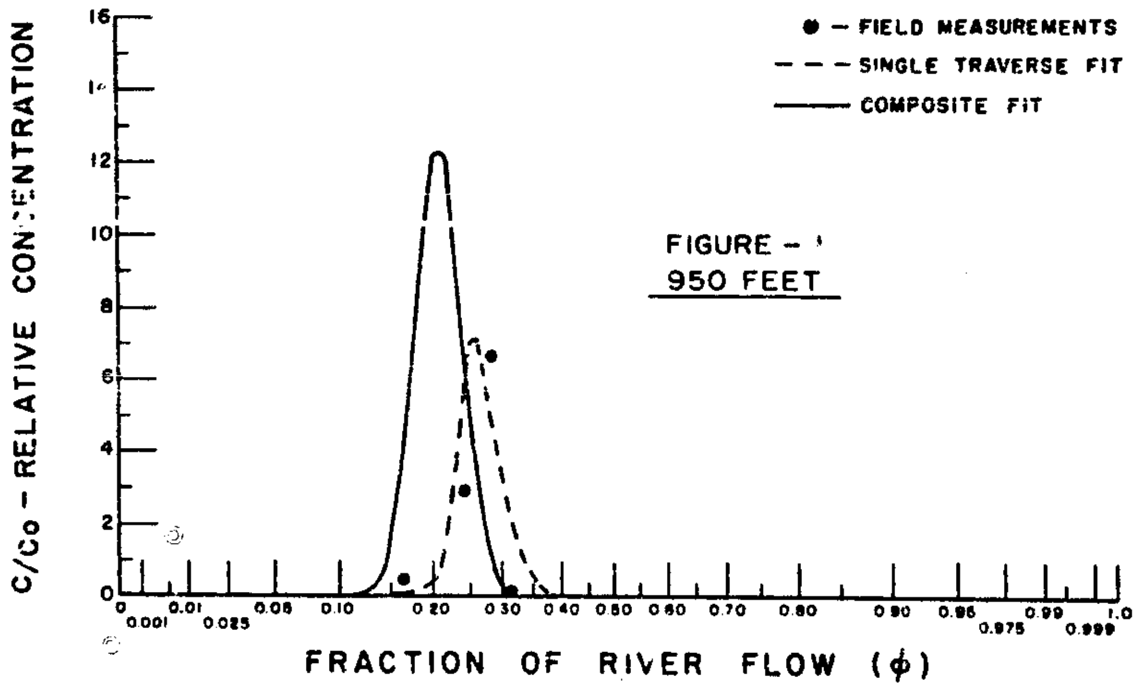
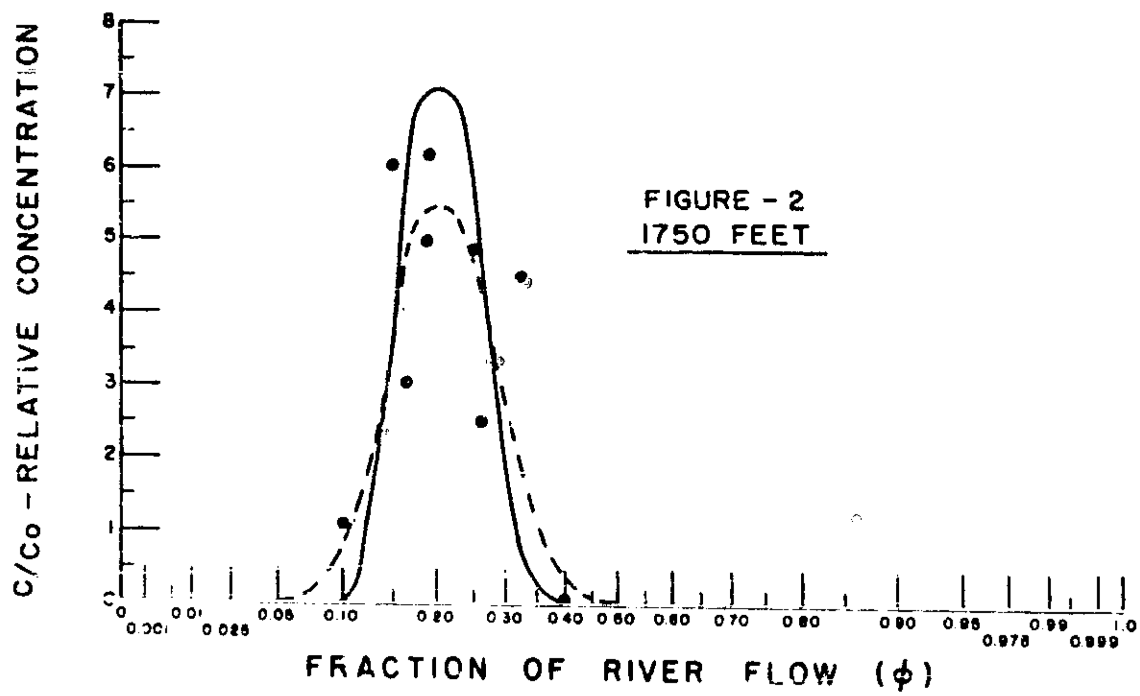
ACTIVITY DENSITY PATTERN AT SELECTED DISTANCES DOWNSTREAM FROM THE DISCHARGE POINT





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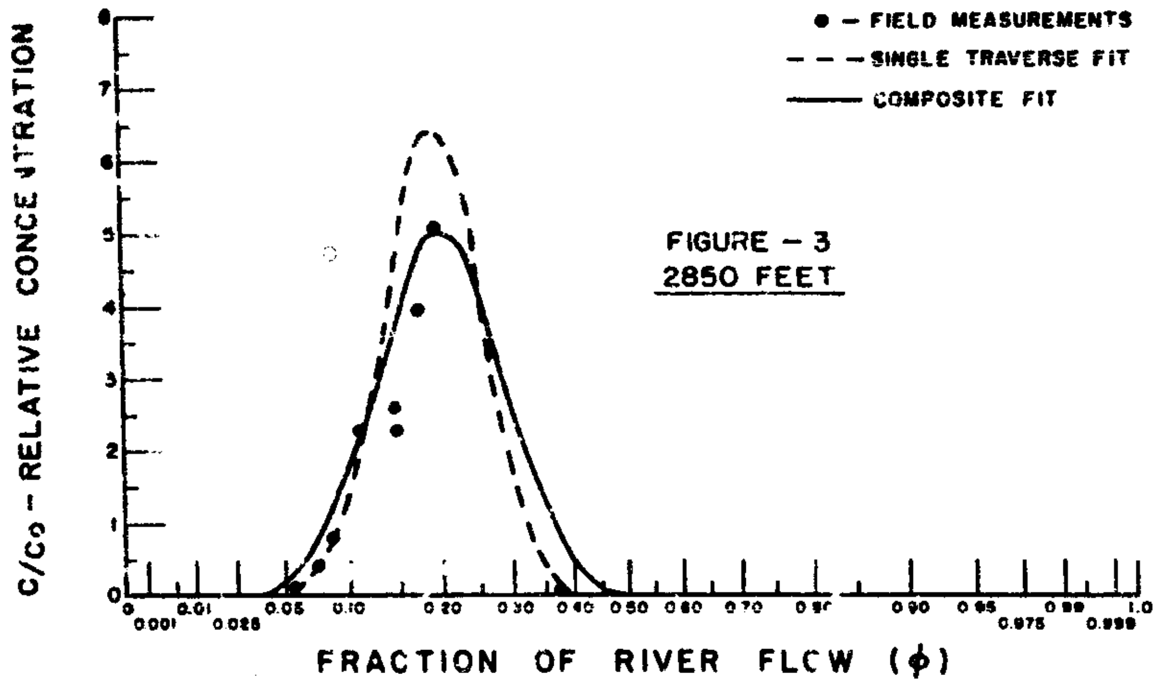

FIGURE – 3
2850 FEET

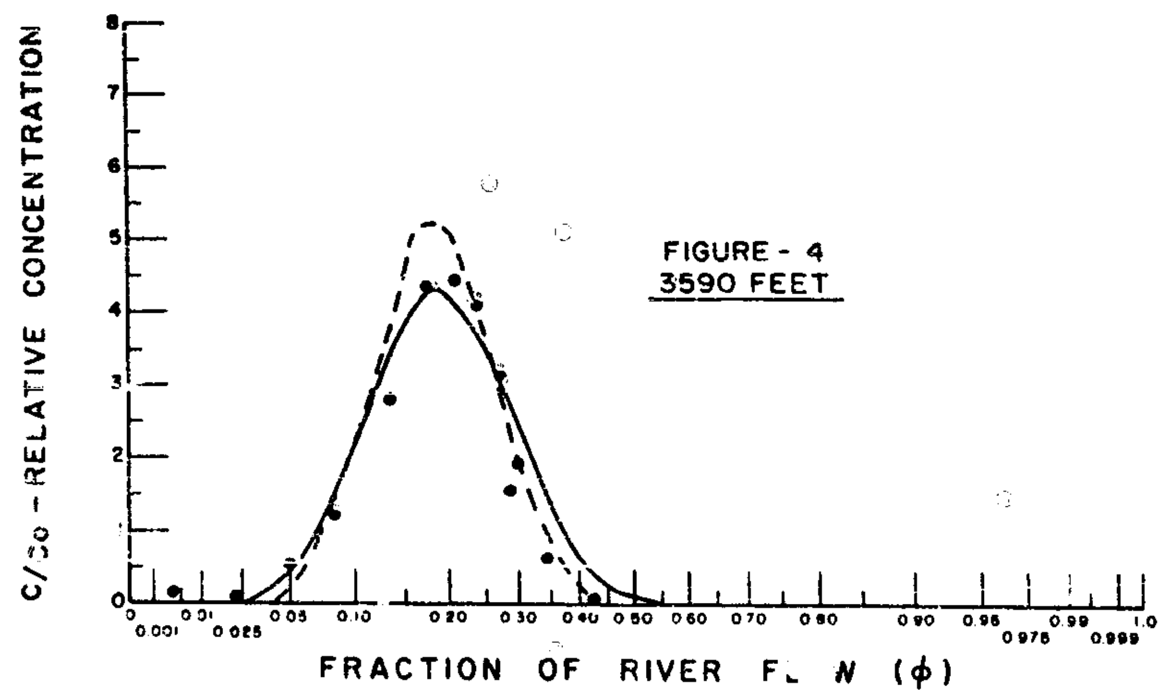

FIGURE - 4
3590 FEET

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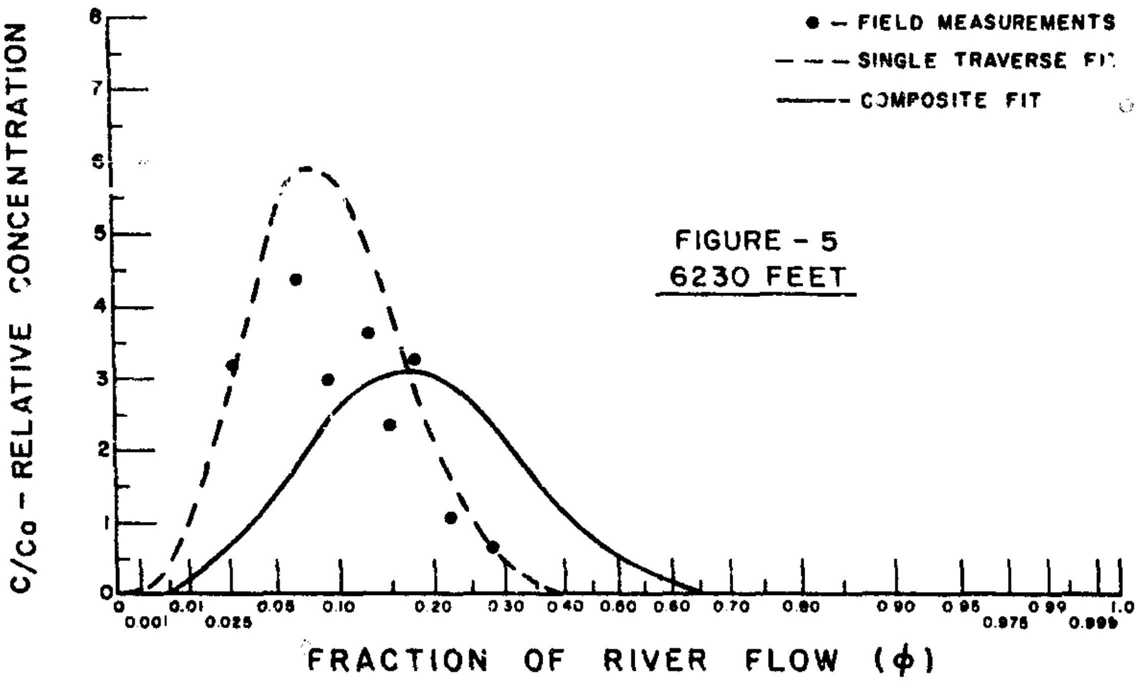

FIGURE – 5
6230 FEET

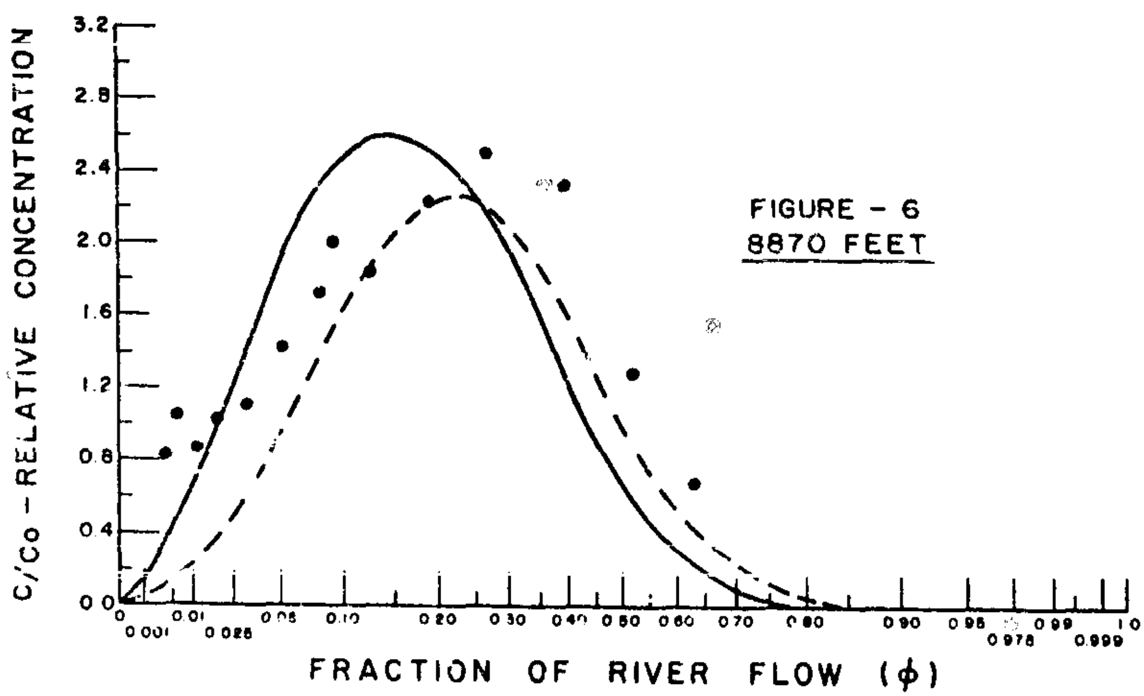

FIGURE – 6
8870 FEET

ACTIVITY DENSITY PATTERN AT SELECTED DISTANCES DOWNSTREAM FROM THE DISCHARGE POINT

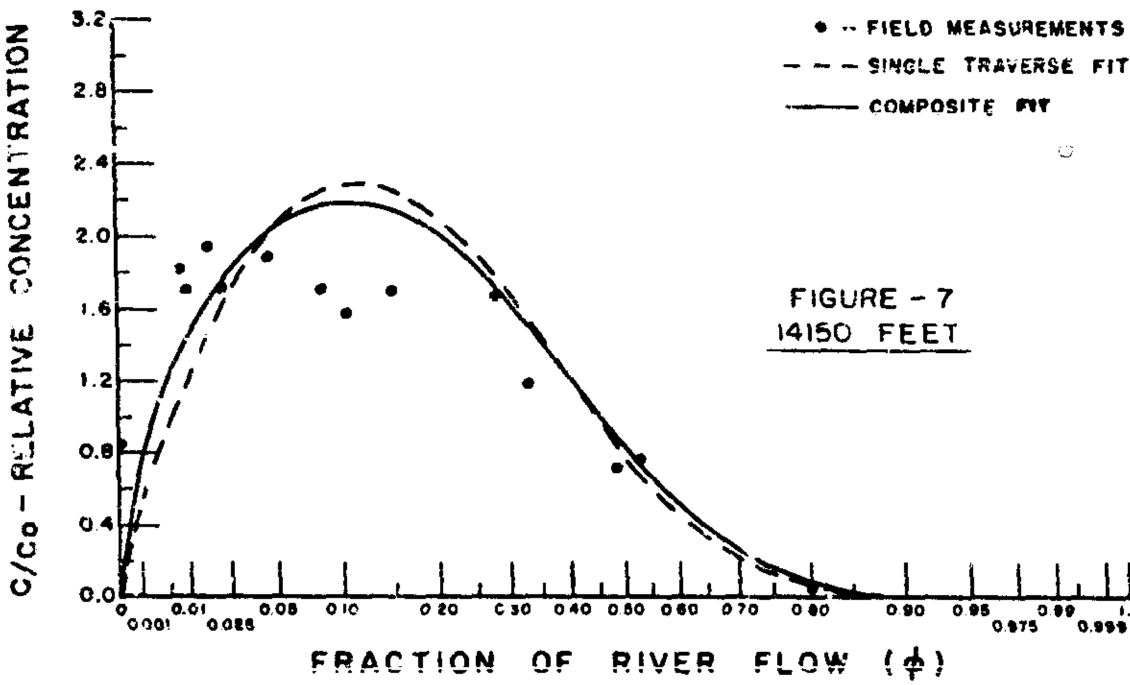

FIGURE - 7
14150 FEET

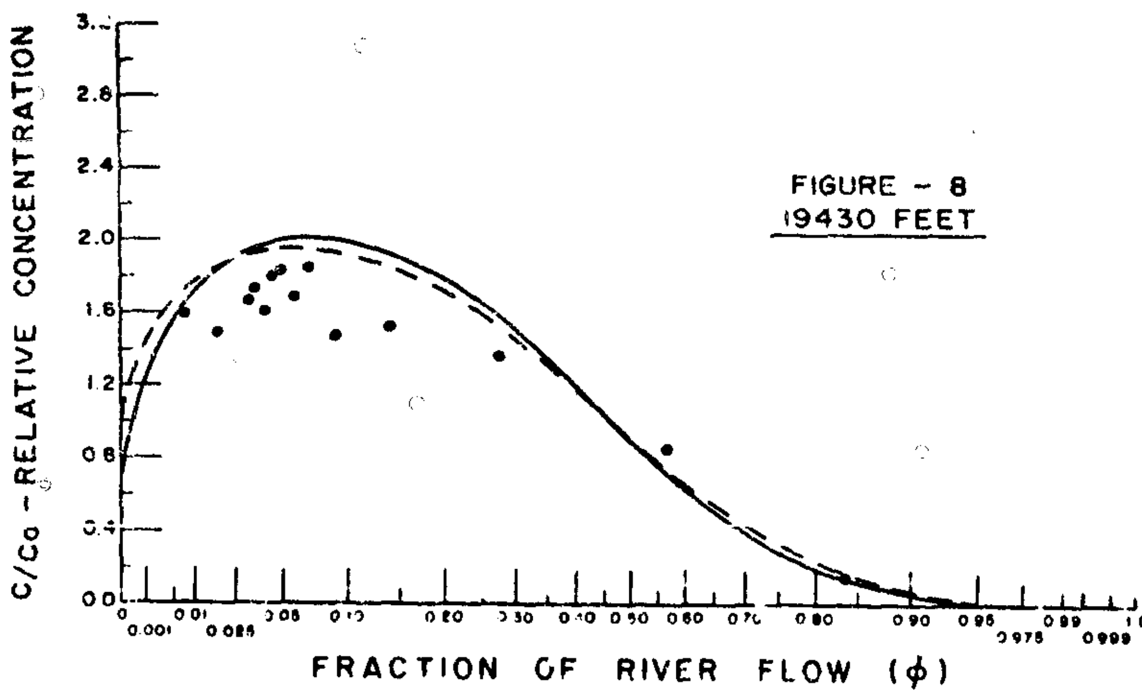

FIGURE - 8
19430 FEET

ACTIVITY DENSITY PATTERN AT SELECTED DISTANCES DOWNSTREAM FROM THE DISCHARGE POINT